

Difficulty Analysis for Learners in Problem Solving Process based on the Knowledge Map

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Abstract

This paper proposes a methodology to calculate both the difficulty of the basic problems and the difficulty of solving a problem. The method to calculate the difficulty of problems is according to the process of building a problem, including *Concept Selection*, *Unknown Designation*, and *Proposition Construction*. Some necessary measures observed in the problem building process are also defined in this paper in order to formulate and calculate the *difficulties*. Beside the difficulty of the basic problem, four difficulty dimensions, *Identification*, *Elaboration*, *Planning*, and *Execution*, corresponding to the problem solving steps are also analyzed in this paper. With these difficulty measures learners can understand what kinds of problems they met and what sorts of problem solving strategies are emphasized in solving the problem. To reach our goal, an *Item Generating System* is constructed for demonstrating and supporting the difficulty calculation in the end of this paper.

1. Knowledge and Problem Analyses

When suitable knowledge structure is designed, the CAI system can be used to proceed with tutoring, solving problems, or misconceptions diagnosing. The knowledge structure, which used here, is *Knowledge Map*. *Knowledge Map* consists of two major parts, *Concept Hierarchy* and *Concept Schema* that analyzed by using *Knowledge Object*. [6][10] Using the *Knowledge Map*, a problem solving process is constructed as followed. Problem solving has already been discussed for a long while since 1910. [2][3][9] It is a problem when someone has a goal but the goal is blocked for lack of information resources. [5] Two problem structures, *Problem Graph* and *Problem matrix*, can be displayed to illustrate this definition. [7]

Based on the previous definition, basic problem architecture has three major parts: *Sub-Problem Flag Attribute*, *Given Attributes* and *Unknown Attributes*, which *Sub-Problem Flag Attribute* is presented by *Core Knowledge Object*. Using the problem architecture discussed above, the steps of constructing a problem need

three major steps: *Concept Selection*, *Unknown Designation*, and *Proposition Construction*. [7]

2. Problem Difficulty Features

Each step of constructing a problem mentioned in previous section decides its difficulty. Ignoring learners' reading ability, this paper only focuses on the first two steps of problem construction. In *Concept Selection*, *number of sub-problems* (γ_{sub_prob}) apparently influences the difficulty of the problem. *Number of Needed Attributes* (γ_{need_attr}) indicates the quantity of attributes needed to learn, which influence the next two features, *Learning Sequence* (γ_{learn_seq}) and *Concept Depth* (γ_{cpt_depth}). *Learning Sequence* is a difficulty feature that indicating learning order for learners and *Concept Depth* shows the specialization of the concept.

The next step, *Unknown Setting*, affects all attributes described in the problem. *Number of Unknown* ($\gamma_{unknown}$) is one difficulty feature usually used in measuring problem. *Number of Given Attribute* (γ_{given_attr}) indicates the information this problem supports. *Number of Elaborating Attributes* (γ_{elb_attr}) shows the hidden information cover up in the problem. Attributes setting also influence *Mathematical Complexity* (γ_{math_cpx}), which can be another difficulty feature of the problem. This paper focuses on basic problem and disregard knowledge analysis of mathematics. *Number of Sub-Problems* and *Mathematical Complexity* will be omitted in the following (article) section.

3. Difficulty Calculation

Disregarding the *Number of Sub-Problems* and *Mathematical Complexity* features, some needed measures' definitions are listed below and most of them are related to the *Knowledge Map*.

#given_attr:	number of given attributes in the problem
#unknown:	number of unknown attributes in the problem
:	the root of <i>Knowledge Map</i>
θ_i :	<i>i</i> th concept
CS(θ_i):	<i>Concept Schema</i> of <i>i</i> th concept
size(CS(θ_i)):	number of attributes in concept from full <i>Concept Schema</i>
height(θ_i):	the height of sub-tree from <i>i</i> th concept
attr _{LS} (CS(θ_i)):	learning sequence attribute stored in <i>Concept Schema</i>
attr _{manip_cpt} (θ_i):	attributes extracted to manipulating concepts in

Problem Matrix	
$size(attr_{manip_cpt}(\theta))$:	number of manipulating concepts in Problem Matrix
$attr_{max_unknown}(\theta)$:	maximum number of unknown in i th concept

Using these measurement definitions above, each feature of difficulties can be formulized as below:

$$\begin{aligned} \gamma_{need_attr} &= size(CS(\theta)) / \arg \max_{j \in KM} size(CS(\theta_j)) \\ \gamma_{learn_seq} &= attr_{LS}(\theta) / \arg \max_{j \in sibling(\theta)} attr_{LS}(\theta_j) \\ \gamma_{cpt_depth} &= (height(root) - height(\theta)) / height(root) \\ \gamma_{given_attr} &= (size(attr_{manip_cpt}(\theta)) - \#given_attr + 1) / size(attr_{manip_cpt}(\theta)) \\ \gamma_{unknown} &= \#unknown / attr_{max_unknown}(\theta) \\ \gamma_{elb_attr} &= (size(attr_{manip_cpt}(\theta)) - \#given_attr - \#unknown + 1) / \\ & \quad size(attr_{manip_cpt}(\theta)) \end{aligned}$$

4. Problem Difficulty Dimensions

Though the paper analyzes difficulty features corresponding to the problem structure, learners cannot use them easily to understand what ability they get wrong in solving problem. This paper use problem solving analysis to design four types of difficulty for problem solvers realize the characteristics of the problem: *Identification Difficulty* (γ_{idf}), *Elaboration Difficulty* (γ_{elb}), *Planning Difficulty* (γ_{pln}), and *Execution Difficulty* (γ_{exc}). These four difficulty dimensions come from the four steps of problem solving in the previous study and all the values of the difficulty dimensions come from the setting of difficulty features. [1][4][6][8][10] To transform difficulty features to difficulty dimensions, the transform formulas are listed below: (ω_{ij} indicates customized weight)

$$\begin{aligned} \gamma_{idf} &= \omega_{21} * \gamma_{learn_seq} + \omega_{31} * \gamma_{cpt_depth} \\ \gamma_{elb} &= \omega_{12} * \gamma_{need_attr} + \omega_{42} * \gamma_{given_attr} + \omega_{62} * \gamma_{elb_attr} \\ \gamma_{pln} &= \omega_{53} * \gamma_{unknown} + \omega_{63} * \gamma_{elb_attr} \\ \gamma_{exc} &= \omega_{54} * \gamma_{unknown} \end{aligned}$$

To demonstrate the usage of difficulty dimension, an enhanced *Item Generating System* based on [7] is constructed. Figure 1 shows an example of generated problem. This problem has one unknown and three given attributes. In the information from *Knowledge Map*, there are eleven attributes in *Concept Schema* of "Motion of Constant Acceleration"; seven of them are manipulating concept and three of them are manipulating relations. According to these settings, six difficulty features and four difficulty dimensions are listed below:

$$\begin{aligned} \gamma_{need_attr} &= 11 / 13 = 0.8 & \gamma_{given_attr} &= (7 - 3 + 1) / 7 = 0.7 \\ \gamma_{learn_seq} &= 2 / 2 = 1 & \gamma_{unknown} &= 1 / 2 = 0.5 \\ \gamma_{cpt_depth} &= (4 - 2) / 4 = 0.5 & \gamma_{elb_attr} &= (7 - 3 - 2 + 1) / 7 = 0.4 \\ \gamma_{idf} &= 0.5 * 1 + 0.5 * 0.5 = \underline{0.75} \\ \gamma_{elb} &= 0.3 * 0.8 + 0.3 * 0.7 + 0.4 * 0.4 = \underline{0.61} \\ \gamma_{pln} &= 0.5 * 0.5 + 0.5 * 0.4 = \underline{0.45} \\ \gamma_{exc} &= 1 * 0.5 = \underline{0.5} \end{aligned}$$

4. Conclusion and Future Works

This paper designs an *Item Generating System*, which constructs four difficulty dimensions for helping learners understanding the emphatic problem solving technique belongs to which problem-solving step. The future work of this paper is to solve the difficulty calculation of complex problem, which influences the *Number of Sub-Problems* in difficulty features. *Mathematical Complexity*

also affects the difficulty calculation. It can also become an interesting issue for discussion.

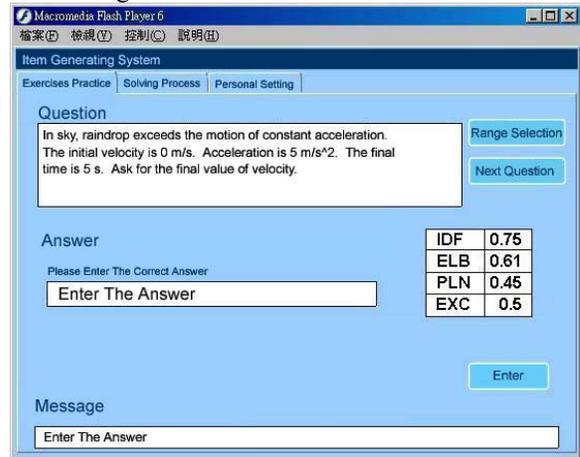


Figure 1. Item Generating System

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